Preconditioned Reflection Full Waveform Inversion for Subsalt Imaging

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Summary

Reflection full waveform inversion has become an effective tool to generate low-wavenumber deep model update using reflected energy and to improve seismic imaging, especially in the subsalt area. However it is still a challenging inverse problem that suffers from local minima issues. To tackle this problem, we propose a new weighted traveltime based objective function with the correction of the kinematic errors between the synthetic and real data, instead of simply minimizing the data residual for least squares objective function. Therefore it helps avoid cycle skipping issues to overcome some of the problems with local minima and relaxes the requirement for successful inversion. Additionally, we propose a preconditioned optimization scheme to mitigate the artifacts from the kernel of reflection full waveform inversion and improve convergence rate with better results. We compare synthetic examples to demonstrate the benefits. The field example shows successful low wavenumber update from reflections with improved subsalt images.
Introduction

Conventional full waveform inversion (FWI) has been an essential tool to generate high-fidelity earth models for better seismic imaging and structural interpretation. It solves a least squares problem by minimizing the misfit between the acquired and synthetic data (Tarantola, 1984). Considering the intrinsic problem for conventional FWI, the success in providing reliable background models heavily relies on the offset range and low frequency content in the acquired data. With limited offset range and low frequency data, reliable background updates are usually constrained by the depth penetration from refraction energy with only shallow improvements to the model.

We have to make use of reflection data in order to obtain a deeper update for a smooth background model. However using reflection data with conventional FWI while missing low-wavenumbers, only the high-wavenumbers of the target model can be inverted due to the dominant contribution from the migration kernel on the tomographic gradient. A different objective function that can separate the tomographic from the migration contribution is required for a successful reflection FWI. Various approaches have been proposed to use the reflection energy. Van Leeuwen and Hermann (2013) introduced a penalized objective function for FWI with a reconstructed wavefield method. By reconstructing an extended source to generate reflections, the computed gradient with respect to velocity model results in a deeper low-wavenumber update with proper choice of penalty parameter (Wang et al., 2016). Alternatively, we can also use a density or reflectivity model to generate reflections, and then formulate an objective function as a function of the smooth background model (Xu et al., 2012). In this paper, the Born approximation is used to generate reflection data while the reflectivity model is updated for each iteration.

Inverting reflections with FWI is a highly nonlinear problem and convergence to local minima is a challenge. A limitation for inverting reflections then becomes the need for good starting models. Over the last decade, various efforts have been made to mitigate the problems of local minima and many alternative methods have been proposed (Shen and Symes, 2008; Van Leeuwen and Hermann, 2013; Biondi and Almomin, 2014; Warner and Guasch, 2016; Wang et al., 2016; Huang et al., 2016; Vigh et al., 2016; Luo et al., 2016). All these previous works and our proposed method in this paper aim to avoid convergence to local minima by adding additional parameters to the model and expanding the search space in the hope that eventually the non-physical model converges to a physical one. By adopting an objective function that automatically adjusts for the poorly matched events, our approach is less likely to cycle-skip and in the meanwhile provides a deeper background model update by using reflected energy. We propose a preconditioned optimization algorithm to suppress artifacts introduced by reflection FWI and thereby increase the convergence rate.

Method and Theory

We first split the velocity model into a long wavelength component $m$ and a short wavelength component $r$ and apply the first order Born approximation to the acoustic wave equation. Then we have the following duo-propagator for Born modeling (Hudson and Heritage, 1980)

$$\Box [m] u = f,$$
$$\Box [m] \delta u = r \partial^2_t u. (2)$$

where $m$ represents the subsurface smooth background velocity model, $r$ represents the reflectivity model, $\Box [m]$ is the wave operator or D’Alembert operator, $u$ is the forward propagated transmitted wavefield and $\delta u$ is the forward propagated reflected wavefield, $f$ is the source function. Let $S[m]$ denote the solution operator for the above linearized forward propagated wave equation (1). Then we can compute the forward transmitted wavefield $u$ as $u = S[m]f$, and the forward reflected wavefield $\delta u$ as $\delta u = S[m]r \partial^2_t S[m]f$. Given an initial background velocity model and a reflectivity model, our proposed objective function optimizes over smooth background model to minimize the local travel time error in the data. It emphasizes the kinematics and is implemented in the time domain using high-order finite difference scheme. The forward modeled data are obtained from solving the wave equations with a first order Born approximation and time shift traces are computed from localized cross-correlation between the acquired field data and the predicted synthetic data. Luo and Schuster (1991) introduced the travel-
time based objective function. We modified it so that our time error becomes a local measurement which
is a function of time and space. Our modified version can be beneficial when a single time-shift per trace
is not adequate for handling complicated scenarios. Our proposed optimization problem is
\[
\min_m \ J[m] = \frac{1}{2} \| A \tau_0 \|^2_2, \\
\text{s.t. } \tau_0(x_r,t;x_s) = \text{argmax}_\tau \ c(\tau, x_r, t; x_s),
\]
where the windowed cross-correlation function \( c(\tau, x_r, t; x_s) = \int_{s_0}^{s_1} P \delta u(x_r, t + s; x_s) d_0(x_r, t + \tau + s; x_s) ds \).
Here \( A \) is the weighting operator, \( P \) is the restriction operator (a projection) that records the forward
modeled wavefield at the receiver locations, \( d_0 \) is the observed field data, and \( P \delta u \) is the Born modeled
data for the current model \( m \). \( \tau \) is a time lag and \( \tau_0 \) is the optimal time lag function at time \( t \), receiver \( x_r \),
and source \( x_s \), computed from cross-correlation from \( t + s_0 \) to \( t + s_1 \). From implicit differentiation and the
midpoint rule, the new approximate adjoint source \( R \) can then be computed as follows
\[
R(x_r,t;x_s) = -\frac{(s_1 - s_0) A^T \delta \tau_0 \delta d_0(x_r,t + \tau_0;x_s)}{\int_{s_0}^{s_1} P \delta u(x_r, t + s; x_s) \frac{\partial^2 d_0}{\partial \tau^2}(x_r, t + \tau + s; x_s) ds}.
\]
Note that \( \boxtimes = I \), the gradient for our proposed objective function with respect to the long wavelength
component \( m \) can be calculated using
\[
\nabla_m J \approx \frac{2}{m^2} \langle \partial_t^2 \delta u, S^* P^* R \rangle + \frac{2}{m^2} \langle \partial_t^2 u, S^* r^2 P^* P^* R \rangle.
\]
As we noticed, the tomographic kernel from reflection FWI may introduce strong artifacts from the "rabbit ears" and also a sharp velocity contrast does not satisfy the Born assumptions. We design a
preconditioner \( S \) by the change of variables \( \tilde{m} = Sm \), then the conjugate gradient (CG) descent direction
for preconditioned model \( \tilde{m} \) at \( k + 1^\text{th} \) iteration can be modified as follows
\[
\tilde{m}_{k+1} = \tilde{m}_k + \alpha_k d_k, \\
d_{k+1} = C g_{k+1} + \beta_k d_k,
\]
where \( C = SS^T \), \( \beta = \frac{s_1}{s_0} \frac{C_{g_{k+1}}}{C_{d_k}} \), \( \alpha_k \) is the steplength, and \( g_k \) is the steepest descent direction. The pre-
conditioned CG improves the convergence rate for reflection FWI with less artifacts and better updates.
Since our preconditioned reflection FWI utilizes a kinematic based objective function, which quanti-
fies local move out errors between reflection events, it reduces the risk of converging to local minima
with an inaccurate initial model, compared with those using the conventional data misfit norm, and
provides a deeper reliable background update with reflected energy and higher convergence rate. This
approach is not limited to reflection FWI, it is also an effective method for mitigating the problem with
cycle-skipping and increasing convergence rates for conventional FWI. Similarly we may replace the
data residual for least squares FWI with the same adjoint source as explained here for formulating the
corresponding gradient.

Synthetic and Field Examples
We now demonstrate our method on a 2D synthetic example. Field data were generated with a Born finite
difference method with a single reflector at 5000 m depth and constant background velocity of 2000 m/s
with a single source receiver pair. Our first initial model had an incorrect constant velocity 2200 m/s.
In contrast to descent direction of least squares (LS) reflection FWI in Figure 1(a), the preconditioned
reflection FWI in Figure 1(b) produces the correct direction for updating the model. Figures 1(c) and
1(d) show the case where the starting model is slower by 10% than the true model. Comparing with the
descent direction for least squares reflection FWI shown in Figure 1(c), the preconditioned reflection
FWI in Figure 1(d) provides us with a much better descent direction that will lead to faster convergence.

Next we demonstrate the advantage of the proposed approach in the Marmousi dataset. The initial model
shown in Figure 2(a) is a simple gradient model. The true model is shown in Figure 2(b), and was used to
generate the field data set comprised of 59 common shot gathers with a shot spacing of 200 m. Each shot gather contains 241 receivers with an interval of 20 m. The lowest frequency used for inversion was 3 Hz and the maximum offset was 4800 m. Figure 2(c) shows the inversion result from least squares (LS) conventional FWI while missing a good starting model and low frequency data. Although the model captures some of the true velocity trend, it converged to a local minima with incorrect layer velocities. Our preconditioned reflection FWI generated reasonable updates however it has not converged to the true model. In contrast it produced a less detailed background model that can be used as a more accurate starting model for conventional FWI. Using inverted result from preconditioned reflection FWI as a starting model, conventional FWI converged to a more correct model shown in Figure 2(e).

We finally present another application to a 3D streamer data set. This narrow azimuth seismic survey was located in the Campeche area offshore Mexico. The maximum offset was 6200 m and the lowest frequency used for inversion was 3 Hz. Figure 3(a) shows the initial velocity that was used for inversion with a maximum depth of 10000 m overlaying the initial stack image and Figure 3(b) shows the inverted model from preconditioned reflection FWI with the updated stack image. The image using the inverted model in Figure 3(b) shows improvements at the base salt and subsalt region indicated in the highlighted areas of the images with respect to Figure 3(a). Comparing the modeled shot gathers overlaying the field shot gathers, our method automatically aligned the events to avoid cycle-skipping with an improved fit around the first arrivals shown in Figure 3(d) and 3(e). The jitter in the shifted final modeled data (Figure 3(e)) is caused by areas of low S/N in the field data and will have little influence on the final result due to the application of the preconditioner. Preconditioned reflection FWI can provide a better reference model for FWI and a deeper update such as in subsalt GOM scenarios.

Conclusions

We presented the theory and applications of our proposed inversion method, preconditioned reflection FWI. We formulated a new objective function that is based on the weighted traveltime shifts with the correction of the kinematics errors. Therefore it helps avoid cycle skipping issues to overcome some of the problems with local minima and relaxes the requirement for successful inversion. Additionally, we
proposed a preconditioned optimization scheme to mitigate the artifacts and improve convergence rate. The results showed successful low wavenumber update from reflections with improved subsalt images.

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References