Reconstructed Full Waveform Inversion with the Extended Source
Chao Wang, David Yingst, Paul Farmer, Ian Jones, Gary Martin, and Jacques Leveille, ION

SUMMARY

Conventional full waveform inversion (FWI) has been extensively applied to real seismic data and has successfully generated high-fidelity earth models for better seismic imaging and structural interpretation. Considering the nonlinearity and ill-posedness of the problem for conventional FWI, the success in providing reliable updated models heavily relies on the accuracy of the initial models and low frequency contents in the acquired data.

To relax the requirements of good initial models and adequate low frequencies, we propose a novel approach to time domain reconstructed full waveform inversion with the extended source (RFWI). RFWI relaxes the constraint that the forward modeled data exactly solve the wave equation as in conventional FWI, and instead uses an $\ell_2$ approximate solution. RFWI estimates earth models and jointly reconstructs an extended source by minimizing an objective function that penalizes the wave equation error while fitting the data. RFWI extends the solution space and therefore overcomes some of the problems with local minima that prevent conventional FWI from obtaining a reliable solution with an inaccurate starting model and/or insufficient low-frequency data.

INTRODUCTION

Conventional full waveform inversion (FWI) has been an essential tool to build high-fidelity earth models by minimizing the misfit between the acquired and modeled data (Lailly, 1983; Tarantola, 1984; Virieux and Operto, 2009). This least-squares problem has been implemented in both the time and frequency domain (Sirgue and Pratt, 2004; Wang et al., 2013). However, it is a highly nonlinear, ill-posed inversion problem and mitigating convergence to local minima is a severe challenge. The greatest limitation of FWI that affects the success in generating reliable solutions for large-scale production jobs is the critical requirement of low-frequency data coupled with a good starting model.

Over the last decade, various effort has been made to mitigate the problems of local minima and many alternative methods have been proposed (Shen and Symes, 2008; van Leeuwen and Hermann, 2013; Biondi and Almomin, 2014; Warner and Guasch, 2016; Wang et al., 2016; Huang et al., 2016). All these previous works and our proposed method in this paper aim to avoid convergence to local minima by adding additional parameters to the models and expanding the solution space.

We now focus on the time-domain method and implementation using finite difference scheme. To compute the misfit function for time-domain FWI, the conventional forward modeled data are extracted from the source wavefield generated by solving the forward wave equation exactly with the given source signature. Our reconstructed forward modeled data are obtained from the reconstructed source wavefield by solving the wave equation with the extended or reconstructed source. We refer to this method as reconstructed full waveform inversion with the extended source (RFWI). Having simulated the forward modeled data and source wavefield, conventional FWI searches for earth models such that the synthetic data have the best match to the field data in the least-squares sense. RFWI optimizes over earth models and the source wavefield jointly to minimize the data misfit subject to the wavefield being consistent with the wave equation in an $\ell_2$ sense. By reconstructing a better source wavefield from the extended source instead of the original source signature with the current model, RFWI relaxes the severe requirement for conventional FWI and provides a more reliable model update.

By including the source wavefield as an additional parameter to the search space, RFWI adds the wave equation error as a penalty term to the original data misfit in conventional FWI and formulates a joint minimization problem. We reconstruct the source wavefield and estimate the earth models in an alternating fashion. We first reconstruct the extended source by minimizing the wave equation error together with the data misfit. It is estimated from solving the normal equation which is equivalent to the least-squares solution. The source wavefield is then reconstructed from forward propagation of the extended source. With the reconstructed wavefield and the extended source, models are updated with a gradient based optimization method and inverted models are used for reconstructing another source and wavefield at next iteration.

Time-domain implementation differentiates RFWI from other previous works that are related to wavefield reconstruction inversion in the frequency domain (van Leeuwen and Hermann, 2013; Huang et al., 2016) and provides a more suitable solver for processing 3D large-scale production data sets.

By adding additional parameter and expanding the search space, RFWI reconstructs the forward modeled data to better fit the field data and avoid cycle skips. Therefore it mitigates some of the problems with local minima with inaccurate initial models and/or inadequate low-frequency contents that limit the success of conventional FWI. While FWI usually relies on diving waves, RFWI takes advantages of reflected waves from wavefield reconstruction and produces deeper model updates. It also compensates the wavefield errors that relates to the acoustic assumptions and approximations during the wave propagation. From the observations of both synthetic and field examples, RFWI demonstrates more advantages in areas with sharp velocity contrasts, especially with the presence of the salt.

This paper first presents the theory and methodology for 3D time domain RFWI. It also discusses the differences and similarities between conventional FWI and RFWI. The benefits of RFWI over conventional FWI are demonstrated using a 2D synthetic example. Finally, the applicability of RFWI on field data is illustrated on both 2D and 3D streamer data from offshore Mexico.
Reconstructed Full Waveform Inversion with the Extended Source

THEORY

Consider the following general wave equation,
\[ \Box[m]u = f, \]  
(1)
where \( m \) represents the subsurface model parameters, \( \Box[m] \) is the wave operator or D'Alembert operator, \( u \) is the forward propagated wavefield, and \( f \) is the source signature. Let \( S[m] \) denote the solution operator of the forward propagated wave equation (1). At each iteration, conventional FWI solves the wave equation exactly with the given source and the current model to obtain the source wavefield \( u = S[m]f \). The least-squares problem for reconstructing the source wavefield \( u \) is used to reconstruct the source wavefield \( \delta f \) which is used to reconstruct the source wavefield \( \delta \) and \( \delta \) is used to reconstruct the source wavefield \( u \). Note that \( \delta \) is the velocity model that will be inverted for, while anisotropy parameters are fixed. The velocity model \( v \) then can be updated using a conjugate gradient method and the gradient for the objective function (4) w.r.t. \( v \) can be calculated using

\[ \nabla_v J_k[g(v), v] \approx \nabla_v J_k[g(v), v] \]

\[ = \begin{cases} \frac{2}{v^3} \partial_t^2 S \delta \delta f, \quad S' P' d_0 - S' P' S \delta f \\ \frac{2}{v^3} \partial_t^2 \delta S, \quad S' P' d_0 - S' P' S \delta f \end{cases} \]

This gradient computation can be easily extended to more general wave equations and used to perform multi-parameter inversion for velocity and other earth models, such as anisotropy parameters, attenuation quality factor, and/or density, either simultaneously or sequentially. When the penalty factor \( \lambda \) is large enough, RFWI and conventional FWI converge to very similar results. Thus the penalty factor has to be chosen carefully to make RFWI produce favorable results and it varies with iterations. The second term in the gradient computation may be ignored to reduce the computation cost.

SYNTHETIC EXAMPLE

We first demonstrate the advantage of RFWI by applying it to a 2D synthetic data set and draw a comparison with conventional FWI. The true model is a modified SMAART Pluto synthetic salt model as shown in Figure 1(a), which was used to generate the field data set that has 250 shot gathers with a shot spacing of 20 m. Each shot gather contains 600 receivers with an interval of 20 m. The lowest frequency used for inversion was 4 Hz and maximum offset was 6000 m. The initial velocity model is displayed in Figure 1(b), which is a smoothed version of
Reconstructed Full Waveform Inversion with the Extended Source

Figure 1: Synthetic models

the true model. If we compare the inverted models from 87 iterations of conventional FWI in Figure 1(c) and 54 iterations from RFWI in Figure 1(d), we notice that RFWI provides a more detailed results with faster convergence, especially for the sub-salt area.

FIELD EXAMPLES

The second example involves an application of RFWI to 2D streamer data from offshore Mexico. The acquisition length was 148 km. 745 shots were used with a shot spacing of 200 m. The lowest frequency used for inversion was 3 Hz. Figure 2(a) shows the simple initial velocity model. Figure 2(b) shows the inversion result from RFWI. After the inversion using RFWI, not only the shallow velocity has been updated, but also deeper changes have started building up the top of the salt with an improved salt velocity and refining the deeper structure. We then compare the stack images from the offset gathers to better QC the results. Stack using the inverted model demonstrates better focusing compared to the stack using the initial model as pointed by the red arrows in Figure 3. Finally, we forward modeled the data using the initial and inverted models and generated shot gathers that are displayed in Figure 4(a) and 4(b). Comparing with field data in Figure 4(d) for the same shot record, the forward modeled data using the inverted model after RFWI fit the field data much better than using the initial model, with clear improvement in both the near and far offsets. If we further investigate RFWI results, the reconstructed forward modeled data that are extracted from the reconstructed source wavefield using the RFWI inverted model shown in Figure 4(c) has the best fit to the acquired data.

We finally present an application of RFWI to a 3D streamer data set. This narrow azimuth seismic survey was located in the Campeche area from offshore Mexico. We used 2280 sources with an interval of 500 m. The maximum offset was 6200 m and the lowest frequency used for inversion was 3 Hz. Figure 5(a) shows the legacy velocity model that was used as initial model for RFWI and Figure 5(b) shows the inverted model from RFWI which includes updates for both shallow and deep sections. We then compare the offset gathers to QC the inversion results. Comparing the shallow and top-salts events, the offset gathers using RFWI inverted model shown in Figure 6(b) are more flattened than using the initial model shown in Figure 6(a). Another tool for QC RFWI results is using reverse time migration (RTM). Comparing with RTM image using the initial model in Figure 7(a), image using the RFWI inverted model in Figure 7(b) shows better image focusing and event continuity at the top salt and also at the deeper area below the salt as indicated by the red rectangles. RFWI demonstrates advantages in providing higher-resolution velocity for areas with strong velocity contrasts.
Reconstructed Full Waveform Inversion with the Extended Source

Figure 4: Shot gathers

(a) Using initial model
(b) Using RFWI model
(c) Using reconstruction
(d) Field data

Figure 5: Velocity models

(a) Initial
(b) RFWI inverted

Figure 6: Offset gathers

(a) Using initial velocity model
(b) Using RFWI inverted velocity model

Figure 7: RTM images

(a) Using initial velocity model
(b) Using RFWI inverted velocity model

CONCLUSION

We presented the methods and applications of our proposed novel inversion method - time domain RFWI. RFWI helps avoid cycle skipping issues to overcome some of the problems with local minima and relaxes the requirement for conventional FWI. It demonstrates more advantages in areas with strong velocity contrasts, which makes it a beneficial method for velocity model building with the presence of salt.

ACKNOWLEDGEMENTS

We would like to thank SMAART for providing the pluto synthetic model. The Campeche reimagining program data was reprocessed and reimagined by ION in partnership with Schlumberger, who holds data licensing rights. We also thank ION for permission to publish the results and our colleagues for providing valuable discussion and support.
REFERENCES


