VTI Waveform Inversion with Practical Strategies: Application to 3D Real Data
Chao Wang∗, David Yingst, Robert Bloor, and Jacques Leveille, ION Geophysical

SUMMARY

The term of “Waveform inversion” (WFI) refers to a collection of techniques that use the information from seismic data to derive high-fidelity earth models for seismic imaging. The attractiveness of WFI lies mainly in its lack of approximations, at least in a theoretical sense, in contrast to other model determination techniques such as semblance or tomography. However, a whole raft of approximations must be made to make the technique viable with today’s computing technology and restrictions of seismic acquisition. These are collectively referred to as “waveform inversion strategies” and in this paper we mainly discuss regularization and preconditioning strategies.

WFI is a highly nonlinear, ill-posed problem. As such regularization techniques from optimization theory are beneficial for its solution. Regularization involves introducing additional constraints on the problem usually by restricting smoothness or sharpness of model parameters. These restrictions are driven by a priori geophysical information. In this paper, we apply total variation regularization (TV), a popular method which involves $\ell_1$-norm of the model derivatives. Real data examples show that edge locations of velocity anomalies tend to be preserved using TV regularization. Compared to inverting the Helmholtz operator, time domain implementation is straightforward and relatively fast. Therefore we present the methodology, strategies, and 3D data examples for time domain WFI with TV regularization. We pursue a vertical transverse isotropic (VTI) acoustic formulation which accounts for the effects of anisotropy. Our WFI is a joint inversion for model parameter and source delay time. Model can be parametrized by P-wave velocity and/or anisotropy parameters, $\varepsilon$ and $\delta$. Various preconditioning strategies are also discussed in order to increase the convergence rate for this iterative scheme and reduce the risk of converging to local minima.

This paper presents the time domain acoustic VTI WFI implementation. It also discusses practical strategies for regularization and preconditioning and their influences on the models that are obtained from WFI. These approaches will be illustrated on 3D marine data from the Green Canyon area of the Gulf of Mexico.

INTRODUCTION

Waveform inversion (WFI) is a seismic method for deriving high-fidelity earth models for seismic imaging by minimizing some misfit function of the difference between the field and modeled data. It was first introduced by Lailly, Tarantola, and Mora using a gradient-based iterative algorithm (Lailly, 1983; Tarantola, 1984, 1987; Mora, 1986). Since these pioneering efforts many researchers have attempted to use various strategies and computational schemes to make WFI implemented either in the time or the frequency domain a processing tool for real data sets (e.g. Sirgue and Pratt, 2004; Shin and Min, 2006; Vigh and Sturr, 2006; Operto et al., 2007).

Our WFI has been implemented using two different approaches. One is a time domain approach and the other is a single frequency approach. They were both successfully applied to real 3D data sets (Yingst et al., 2011). Because the wavefields need to be accurately modeled to represent the kinematics of all the waves during WFI iterations, considering the effects of anisotropy often helps to improve WFI results. In this paper, forward modeling and its adjoint computation are based on the time domain VTI acoustic wave equations. Anisotropic WFI shows a more reliable reconstruction of model structure than isotropic WFI and helps to improve WFI results, where the presence of significant anisotropy is included (Plessix, 2009; Gholami et al., 2011). VTI WFI mainly depends on the P-wave velocity, anisotropy parameters, $\varepsilon$ and $\delta$. The definition of a suitable parametrization becomes a key issue for VTI WFI. The sensitivity analysis of acoustic anisotropic WFI has shown that the seismic data are more sensitive to the velocity perturbation than the anisotropy parameters, $\varepsilon$ and $\delta$ (Gholami et al., 2011). Since the gradient is quite sensitive to source time shifts, an accurate source signature delay time is necessary to match the observed and predicted data. In this paper, we focus on the time domain approach and develop a joint inversion for recovering both model parameter (P-wave velocity, $\varepsilon$ and/or $\delta$) and source delay time. The single frequency approach was introduced by Sirgue (Sirgue, 2003). The idea is to propagate the wave in time and accumulate the Fourier transform for one or a small number of frequencies. We also use a complex projection technique which allows for both amplitude and phase matching (Yingst et al., 2011).

WFI is a highly nonlinear, ill-posed problem. As such regularization techniques from optimization theory are beneficial for its solution. Regularization involves introducing additional information in order to solve ill-posed inverse problems. This information is usually in the form of a weighted penalty function, such as restriction of smoothness or sharpness of model parameters. The geophysics of the problem leads to appropriate constraints. In this paper, total variation (TV) regularization is applied. It is a popular method based on the $\ell_1$-norm of the model derivatives. The TV method provides a more useful recovery of velocity profiles from seismic data including a better delineation of discontinuities by avoiding excessive smoothing (Vogel, 2002).

Another concern of WFI is its high computational cost. An iterative scheme based on a preconditioned nonlinear conjugate gradient method is employed in this paper. Preconditioners are useful in iterative methods since the rate of convergence can often be dramatically increased as a result of appropriate preconditioning. We apply muting, offset-weighting, and bandpass filter to precondition the data which lets a user focus the inversion on the most suitable parts of the data. We
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also apply a spectral filter which helps shaping the spectrum of the seismic data and the source wavelet (Lazaratos et al., 2011). This shaping is designed such that the weight of the low-frequency part of the spectrum can be increased. In order to precondition the gradient, a commonly used technique is layer-stripping and/or gradient masking. Other important practical strategies for WFI have also been discussed (Wang et al., 2011). For instance, WFI suffers from cycle skipping when the bandwidth is too broad. This can be mitigated using multi-scale inversion, gradually increasing the frequency bandwidth (Bunks et al., 1995).

This paper presents the time domain VTI implementation with regularization and preconditioning. Then it shows time domain acoustic anisotropic (VTI) WFI results for a 3D real ocean-bottom-cable (OBC) data set from the Green Canyon area of the Gulf of Mexico.

METHODS

Assuming shear velocity being zero, we can derive VTI acoustic wave equations that kinematically models the compressional wave propagation. It involves fourth-order derivatives of the wavefield in time and space (Thomsen, 1986; Alkhalifah, 1998). Based on Alkhalifah (2000)’s pseudo-acoustic approximation, a number of variations of pseudo-acoustic wave equation have been developed (Zhou and Bloor, 2006). Here we use a VTI system of two coupled second-order partial differential equations in terms of P-wave vertical velocity, \( v \), and anisotropy parameters, \( \epsilon \) and \( \delta \):

\[
\frac{1}{v^2} \frac{\partial^2}{\partial t^2} \left( \frac{q}{p} \right) = \left( \begin{array}{cc} 1 + 2\epsilon & \sqrt{1 + 2\delta} \\ \sqrt{1 + 2\delta} & 1 \end{array} \right) \left( \begin{array}{c} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \\ 0 \end{array} \right) \left( \begin{array}{c} q \\ p \end{array} \right) + f, \tag{1}
\]

where \( f \) is the source, \( p \) is the forward-propagated wavefield and \( q \) is the auxiliary wavefield.

The traditional time domain implementation is to minimize the misfit function (Tarantola, 1987):

\[
\min_{m} J_{\delta}(m) = \|d_0 - d\|_2^2, \tag{2}
\]

where \( d_0 = d_0(x_s, t; x_r) \) is the observed seismic data set and \( d = d(x_s, t; x_r) \) is the predicted (i.e. computer simulated) data set for the model \( m \) at source and receiver locations, \( x_s \) and \( x_r \). The predicted data are obtained by sampling the extrapolated wavefield \( p \) generated by a high-order finite difference scheme to the receiver locations, based on the acoustic VTI wave equations (1).

We modify the traditional misfit function slightly and add a TV penalty term:

\[
\min_{m} J_{\delta}(m) = \|T(d_0 - \rho \, d)\|_2^2 + \lambda R, \tag{3}
\]

where \( T \) is a data preconditioner and \( \rho \) is a normalization scalar \( (\rho = \|d_0\|/\|d\|^2) \). Note that the preconditioner \( T \) can depend on frequency bandwidth, spectral filter, offset, and/or time windows. \( \lambda \) is a weighting scalar and \( R \) is the TV regularizer constraints defined as

\[
R(m) = \int \sqrt{\|\nabla(m - m_0)\|^2 + \beta^2} \, dx, \tag{4}
\]

where \( \beta \) is a small value to avoid singularity of the gradient of \( R \) and \( m_0 \) is the initial model.

The misfit function is minimized iteratively by computing the gradient (Tarantola, 1984) at successive models. It uses the gradient at an initial point for an initial direction estimate and updates that direction using the Polak-Ribière implementation of the nonlinear conjugate gradient (CG) method. The line search along the directions uses the BB formula (Barzilai and Borwein, 1988) for an initial estimate of the steplength. BB formula does not require extra forward modeling to evaluate the misfit function and it provides effective initial guess of the steplength for TV regularized problems (Li, 2011). It then applies the backtracking linesearch method to update the steplength.

The gradient of TV constraint term \( R \) is

\[
\nabla_{m} R = \nabla \left( \frac{\nabla(m - m_0)}{\sqrt{\|\nabla(m - m_0)\|^2 + \beta^2}} \right). \tag{5}
\]

The adjoint-state equations of the state equations (1), for computing the back-propagated wavefields \( p^+ \) and \( q^+ \) by back-propagating the residual \( f^+ \), yield

\[
\frac{1}{v^2} \frac{\partial}{\partial t} \left( \begin{array}{c} q^+ \\ p^+ \end{array} \right) = \left( \begin{array}{cc} 0 & 0 \\ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} & 0 \\ 0 & \frac{\partial^2}{\partial x^2} \end{array} \right) \left( \begin{array}{c} 1 + 2\epsilon \left( 1 + \sqrt{1 + 2\delta} \right) \right)^{-1} \right) \left( \begin{array}{c} q^+ \\ p^+ \end{array} \right) + f^+, \tag{6}
\]

In this case, the gradient \( g_{\delta} = \nabla J \) with respect to \( v, \epsilon, \delta \), at the \( k \)-th iteration, are given by

\[
\begin{align*}
\nabla_v J(x) &= \frac{2}{v^3(x)} \sum_{x_s} \sum_{t} \left( \frac{\partial^2}{\partial x^2} p(x_s, t; x_r) q^+(x_s, t; x_r) \right) + \lambda \nabla_v R(x), \\
\nabla_{\epsilon} J(x) &= \frac{2}{v^3(x)} \sum_{x_s} \sum_{t} \left( \frac{\partial^2}{\partial z^2} q(x_s, t; x_r) q^+(x_s, t; x_r) \right) + \lambda \nabla_{\epsilon} R(x), \\
\n\nabla_{\delta} J(x) &= \frac{1}{v^3(x)} \sum_{x_s} \sum_{t} \left( \frac{\partial^2}{\partial x^2} p(x_s, t; x_r) q^+(x_s, t; x_r) \right) + \lambda \nabla_{\delta} R(x).
\end{align*} \tag{7}
\]

The model parameter \( m \) can be \( v, \epsilon \) and/or \( \delta \). Let \( m_0 \) be the initial model, \( m_k \) be the inverted model after \( k \)-th iteration, \( p \) and \( q \) are the wavefields modeled from \( m_k \), and \( p^+ \) and \( q^+ \) are the wavefields obtained by back-propagating the residual

\[
f^+(t) = T(d_0 - \rho \, d),
\]

using the adjoint state equations (6) of the forward acoustic VTI equations (1).
Convergence can be accelerated using gradient preconditioning. The current gradient preconditioning normalizes by the amplitude of the forward propagated wave:

$$g_k^g(x) = \frac{g_k(x)}{\sqrt{\sum_{x_i} \sum_{p(x_i)} p(x_i,x_j)^2 + \kappa^2}}. \quad (8)$$

where $\kappa$ is a whitening factor to avoid zeros in the forward modeled data. We also apply layer-stripping and/or masking as another gradient preconditioning tool to accelerate the convergence.

Notice that an accurate estimation of source signature delay time is critical to match the phase between the observed and predicted data. Since the gradient is quite sensitive to source time shifts, we adopt another step to estimate a source delay time before each iteration in order to overcome this problem. This approach is to find an optimal source time delay $\tau$ that maximize the cross-correlation value of the observed and predicted data, i.e.

$$\min_{\tau} J_{s1}(\tau) = \| d_{0\tau} - d \|^2 \quad (9)$$

$$\Leftrightarrow \max_{\tau} J_{s2}(\tau) = d_0 \ast d, \quad (10)$$

where $d_{0\tau}$ is the observed data $d_0$ shifted by a source time delay $\tau$.

Similar equations can be obtained in the single frequency WFI.

**RESULTS**

We present an application of 3D acoustic VTI time domain WFI with TV regularization and preconditioning to 3D marine data. This deep water VSO OBC survey is located in the Green Canyon area of the Gulf of Mexico. The acquisition area was 160 km² and used four-component VSO equipment in deep water (1000+ meters) over relatively shallow salt bodies. Maximum offset used is 7000 m. The lowest frequency in the observed data is about 5 Hz. The source signature was derived from the down-going wavefield on a zero offset section.

The sensitivity analysis of acoustic anisotropic WFI has shown that the seismic data are more sensitive to the velocity perturbation than the anisotropy parameters, $\epsilon$ and $\delta$ (Gholami et al., 2011). Therefore our target model was parametrized by P-wave velocity. Anisotropy parameters, $\epsilon$ and $\delta$, were kept unchanged over WFI iterations. The initial velocity model for WFI, Figure 1(a), came from an anisotropic VTI tomographic inversion.

Various strategies such as multi-scale, layer stripping, and offset weighting are quite useful to minimize the risk of converging to local minima. We applied multi-scale inversion, gradually increasing the frequency bandwidth (Bunks et al., 1995) to avoid cycle-skipping issue. We also applied layer-stripping, inverting successively deeper layers using the presumably correct velocity model in shallower layers to speed up the convergence. Offset-weighting was used to localize the data to some target region to obtain the optimal update and can be automated. The inversion proceeded with spectral shaping applied to the field data and the source wavelet for accelerating convergence. Source delay time and offset weights were automatically computed. We also applied TV regularization in order to preserve the edge of the salt body or any other discontinuity.

The inverted velocity model 1(b) after 5 iterations shows a reasonable shallow update at a depth of about 2000 m including some detailed structures above the salt. The location of the salt body interface is well preserved by TV regularization. The deeper portion of the velocity model was not significantly updated because of limitations imposed by the limited maximum offset in the seismic data. By improving the velocity, the stack gathers 2(b) after the WFI iterations show improvement in flatness compared to the gathers using the initial velocity model 2(a).

To further evaluate the WFI results, we generated stack images with the initial velocity model and the inverted velocity model obtained by WFI. The structures above the salt were significantly improved by comparing the initial stack 3(a) with the updated stack 3(b). We can also clearly see the salt boundary by comparing the initial depth slice at 1950 m 4(a) with the updated one 4(b).
CONCLUSION

In this paper, we presented the methodology, strategies for the acoustic VTI WFI with TV regularization and preconditioning. Considering the effects of anisotropy helps to accurately model the wavefields and therefore helps to improve the WFI results. WFI model can be parametrized by P-wave velocity and/or anisotropy parameters, $\varepsilon$ and $\delta$. Our time domain approach adopts a joint inversion for both model parameter and source delay time. These approaches were illustrated on an application of 3D marine data from the Green Canyon area of the Gulf of Mexico.

From the WFI results, we showed that TV or edge preserving regularization aimed at preserving both the blocky and the smooth characteristics of the velocity model. The preconditioning strategies (spectral shaping etc.) are useful in order to increase the convergence rate.

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EDITED REFERENCES

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