Multi-Region Finite Difference Time Domain Simulation for Airborne Radar
Lei Fu*, University of Connecticut, on leave from Jilin University; Zhao Zhao, University of Connecticut, also at Ion GX Technology; Lanbo Liu, University of Connecticut; Sixin Liu and Linlin Lei, Jilin University

Summary
Traditional finite difference time domain (FDTD) is classic numerical method for solving wave propagation problems such as electromagnetic wave, seismic wave and acoustic wave. As for airborne GPR simulation, modeling space is so large that memory consuming is pretty huge; on the other hand, the area between source antenna and ground surface is homogenous medium – air, where analytic solution could be adopted. We use analytic solution in free space and take use of numerical solution – FDTD in the underground. Different solutions are adopted in different region, and both of them are combined together, that is known as the multi-region FDTD (MR/FDTD) method. The MR/FDTD we discussed here is not the same as before. As we select one face instead of a closed surface as integration surface. The validity of the MR/FDTD method for airborne GPR simulation is investigated by numerical experiments in the work.

Introduction
FDTD is a powerful, robust, and popular modeling algorithm based on the direct numerical solution of Maxwell’s equations in the differential, time-domain form. FDTD can suffer from memory and computational time usage inefficiencies when the computational domain is large, especially when the spacing between scattering objects becomes larger. A large amount of computation time and giant computer memories are devoted to determining the fields at free-space lattice points between objects to provide field continuity. In fact, most of these points are of little or no meaning to the problem at hand. As for airborne GPR detection, the transmitting and receiving antennas are mounted on airplane or helicopter, fly trajectory height of which usually has several wavelength of electromagnetic (EM) wave in free space. In order to decrease the memory consuming and increase the efficiency of the simulation of airborne GPR, we propose a new multi region FDTD (MR/FDTD) method.

Basic Strategy of MR/FDTD
The MR/FDTD method we proposed seeks to avoid the inefficiencies of classical FDTD when applied to airborne GPR modeling problems by eliminating explicit calculation of EM fields in the free-space between the airplane and the ground surface. In airborne GPR modeling problem, the modeling domain could be separated into two parts, free space area and underground geology model area, as shown in Fig. 1. Analytic technic could be used in free space and numerical solution is adopted in underground. Firstly, we should define two surface - incident surface and scattering surface. We usually define the ground surface as incident surface, and one layer above the ground as scattering surface. In Figure 1, the white dash line indicates the incident surface, the source field propagating into the incident surface using analytic technic; the fields in the underground are determined by using localized FDTD lattices; the scattering surface yellow dash line indicated is used to performing a surface integration of equivalent sources - Kirchhoff integration (KIF).

Algorithm of MR/FDTD
The MR/FDTD method combines analytic solution in free space and numerical solution in underground together at each time step. Algorithm flow could be descripted in Figure 2. At each time step:
1) Distribute source wave fields on the incident surface;
2) Update fields in FDTD region;
3) Store field data at scattering surface and the adjacent two layers;
4) Do KIF integration.

Source wave fields propagate from the source position to the incident surface. Assuming the source function could be expressed as \( u(t) \), the distance from the source position to every point of incident surface is \( R \), then the analytic solution of the wave field at incident surface could be expressed using the following formulation (1):
Multi-Region Finite Difference Time Domain Simulation Applied to Airborne GPR

\[ u_j(t) = u(t - \tau_j) / R_j^2 \]
\[ \tau_j = R_j / c \]

where \( c \) is the electromagnetic wave propagating velocity in vacuum, \( \tau_j \) is the time delay, \( u_j(t) \) is the wave field at incident surface.

Figure 2: Algorithm flow chart of MR/FDTD

The concept of KIF has been fully reviewed by Ramahi (1997) and Coleman (2005). As fig. 3 shows, suppose a volume \( V \) containing a source \( J \) enclosed by virtual surface \( S \), and one observation point \( P \) is located outside of \( V \). Suppose the exterior of \( V \) is a homogenous and isotropic medium with dielectric constant of \( \varepsilon \), conductivity \( \sigma \), and magnetic permeability \( \mu \), the geometrical location of \( P \) and the surface \( S \) are known, then the relationship for one way approximation between the values on each point \( Q \) of virtual surface and observation point are fully established by KIF integration formula shown in equation (2):

\[ U(r,t) = \iint \left[ \left( \frac{\nabla U(r',t)}{4\pi R} + \frac{1}{4\pi R} \right) \cdot \left( \frac{\nabla U(r',t)}{4\pi R} \right) \right] dr' ds \]  

where \( \mathbf{n} \) is a normal vector pointed outward on the virtual surface \( S \), \( \mathbf{r} \) is the position vector of \( P \) and \( \mathbf{r}' \) is the position vector of \( Q \) located anywhere on \( S \). \( \tau \) is the retarded time for a signal to be transmitted from \( Q \) to \( P \). Theoretically, an enclosed virtual surface should be properly selected for the integration. But as for airborne GPR simulation, we select the scattering surface as KIF integration surface, and the following parts will test the validity of this method by using numerical experiments.

Validity Test of MR/FDTD

To verify the efficiency and accuracy of the MR/FDTD method, we compare the results from the MR/FDTD to the results form classical Full/FDTD.

Figure 3: Layout of KIF Integration

Figure 4: Two-dimensional slice of the model, red circles represents source positions and white crosses represent receiver positions

The scale of the three-dimensional model we adopted is 60 m, 20m and 44 m in x-, y- and z- direction in Full/FDTD. Spacing interval of grid is 0.2 m. The two-dimensional slice is shown in Figure 4, the ground surface’s z-position is at 2.8 m, the source height to the ground surface is 22.8 m. The relative dielectric constant of underground is 9, and conductivity is zero, there is an abnormal object embed in...
Multi-Region Finite Difference Time Domain Simulation Applied to Airborne GPR

the underground with x- position from 20 to 40 m, 7.2 m beneath the ground surface, thickness of 4 m, with relative permittivity of 81 and conductivity of zero. The source function we used in Full/FDTD and MR/FDTD is Ricker wavelet, the dominated frequency is 50 MHz, time sample interval is 0.25 ns, iterations is fixed the same as 3000 in both method for the purpose of comparing computation efficiency. As for MR/FDTD method, localized FDTD lattices is conducted in the underground, the total grids we need here is only 300x100x100 in the model.

Table 1: Memory and time consumed comparison

<table>
<thead>
<tr>
<th></th>
<th>Full/FDTD</th>
<th>MR/FDTD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memory (MB)</td>
<td>387</td>
<td>193</td>
</tr>
<tr>
<td>Time/Trace (Hour)</td>
<td>1.4</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 1 shows the difference of memory and time consuming between Full/FDTD and MR/FDTD method. The memory consuming of Full/FDTD is about 387 M bytes, while MR/FDTD only needs 193 M bytes, in other words, the memory consumed of MR/FDTD is almost half of which Full/FDTD consumed because the distance of the source to the ground is nearly the same as the depth scale of the underground. The time consumed per trace of Full/FDTD is 1.4 hours, while MR/FDTD only takes 0.6 hour for each simulation. Definitely, MR/FDTD is more efficient compared with Full/FDTD, and the efficiency compared with Full/FDTD depends on the distance between source and the ground.

Synthesis data is recorded at nice different positions from 11 - 51 m by 5 m spacing interval in X- direction as shown in Figure 4, the height of sources and receivers are fixed as -20 m, the offset between source and receiver is 1 m. The received waveform is shown in Figure 5, left result is from Full/FDTD simulation and right result is from MR/FDTD method. In this model, the antenna height is 22.8 m, we can estimate that the two way travel time(TWTT) of reflecting wave from the ground is about 152 ns, which is coincided with simulation results of both method. The distance between ground surface and upedge of the embed abnormal is 7.2 m, the electromagnetic wave propagating velocity is 0.1 m/ns as the underground dielectric constant is 9, we can estimate that the reflecting wave from the upper edge of abnormal should be about 300 ns, this is exactly showed in both results, as the thickness of the abnormal is only 4 m, the reflecting wave from the bottom edge overlaps the reflecting wave form the upper edge. Another interesting phenomenon is that the numerical noise from MR/FDTD is smaller than that from Full/FDTD. The efficiency and accuracy of the MR/FDTD method have been verified from these comparisons.

Forward Modeling of Crevasses Detecting

Crevasses are cracks that form in glaciers and can range from just a few inches across to over 40 feet, and can be over 100 feet deep. Snow covered crevasses are of great danger for travellers, especially if the snow cover is rather thin. Detection of crevasses is critical for safe travelling on glaciers. GPR is a promising tool for the detection of snow covered crevasses, which has been investigated by many field work (Eder et al, 2008). Usually, the relative permittivity of snow is around 1.6, we can estimate the reflection coefficient is about 12%, which means most of the electromagnetic wave energy could penetrate into the glacier. The capability of airborne GPR to detect snow covered crevasses in cold snow and ice has been proved by Arcone (2000).

Figure 5: Received waveform results using two method, (left) from Full/FDTD simulation, direct wave is removed; and (right) from MR/FDTD simulation

Figure 6: A typical crevasse radar image from Shear zone
Multi-Region Finite Difference Time Domain Simulation Applied to Airborne GPR

Figure 6 is typical snow covered crevasses radar image provided by Steven. Reflections from glacier layers are caused by changes in density which is proportional to the permittivity. Diffractions are backscatter from any discontinuity, such as the corner between the wall and a glacier layer, and crevasses (voids) are areas without reflections. We focus on comparing the modeling results between Full/FDTD and MR/FDTD. The 2D model we constructed based on the typical crevasses is shown in Figure 7. The scale in horizontal direction is 60 m, and 30 m in depth. Crevasses covered by snow, crack of the crevasses is about 4 m, the snow/glacier interface is 3-4 m in depth, the relative permittivity of glacier gradually increases from 1.6 to 2.4 as the depth increases. Red circles in Figure 7 represent the source and black crossing is receiver. The offset between source and receiver is 1 m.

Source function is Ricker wavelet, the dominated frequency is 400 MHz, grid size is 0.02 m, time sample interval is 0.04 ns. As for Full/FDTD simulation, the whole model is discreted into 3000 by 2250, while only 3000 by 1500 grids are needed in MR/FDTD simulation.

The simulation result using Full/FDTD is shown in Figure 8 (a), x-axis is horizontal distance, vertical axis is TWTT, the direct wave is removed and auto gain control (AGC) is applied to the profile, we can clearly find the air/snow interface reflecting wave at around 80 ns, and many reflecting waves from glacier layer interface in the later TWTT, these hyperbolas with apexes at around 100 ns are diffraction waves from the crack of crevasse, and diffractions from the discontinuity between glacier layer and crevasse could be identified. All these events got from Full/FDTD are shown in Figure 8 (b) – from the MR/FDTD method. The difference of the two profile are these events from absorbing boundary. In other word, all the features could be simulated using MR/FDTD.

Conclusions

Traditional FDTD is an useful tool for solving electromagnetic wave propagation problem. As for airborne GPR modeling, these area between source antenna and ground is homogenous medium, which means we can use analytic solution such as Green’s function in homogenous instead of FDTD to handle this problem. We combine KIF integration and traditional FDTD together in different area, namely MR/FDTD to handle the simulation problem of airborne GPR. Numerical experiments show that MR/FDTD could get the same accuracy as Full/FDTD. Besides, memory and time consumed of MR/FDTD is much less than which of Full/FDTD.

Acknowledgments

The authors would like to thank Dr. Steven Arcone (CRREL) providing the real data of crevasse radar image. This work was partially supported by the Natural Science Foundation of China (NSFC) through Grant 41074076, and China Scholarship Council.
EDITED REFERENCES
Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2013 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES


