Attenuation compensation in viscoacoustic reserve-time migration
Jianyong Bai*, Guoquan Chen, David Yingst, and Jacques Leveille, ION Geophysical

Summary

Seismic waves are attenuated during propagation. The unwanted attenuation effects in migrated images make interpretation difficult. Based on the dissipation mechanism in a viscoacoustic model consisting of one standard linear solid, we derive a viscoacoustic wave equation in the time domain. The equation includes a pseudo-differential operator for viscosity. We implement lossy acoustic wave equations for the forward and backward wave propagation in order to compensate for the attenuation effects in viscoacoustic reverse-time migration. A numerical test shows that the attenuation effects can be properly compensated. A real data example demonstrates the attenuation compensation in complex geological structures.

Introduction

One of the fundamental properties of wave propagation is the conversion of elastic energy into heat. The anelastic effects described by the quality factor Q can cause amplitude decrease and wavelet distortion. As an example, the presence of low saturation gas in sediments can cause strong attenuation of seismic P-waves. The attenuation effects result in a dimming of migrated amplitudes below the gas. Since the high-frequency components of seismic data are more strongly attenuated than the low-frequency ones, the effects also reduce resolution in the image. Consequently, attenuation compensation is necessary to improve structural imaging and interpretation.

Early efforts to compensate for the seismic attenuation were performed in the unmigrated data domain with an inverse Q-filter (Bickel and Natarajan, 1985; Hargreaves and Calvert, 1991). These methods used one-dimensional reverse wave propagation to create an inverse Q-filter. Since seismic attenuation occurs during the wave propagation, it is necessary to correct the attenuation effects in a migration in order to handle complex geological structures with strong lateral velocity variations and steep dips. Xin et al. (2008) and Xie et al. (2009) performed the attenuation compensation by ray-tracing methods. Because one-way wave-equation migration (OWM) is implemented in the frequency domain, it is natural to incorporate attenuation in imaging. Dai and West (1994), Yu et al. (2002), Wang (2008), and Valenciano et al. (2011) showed different methods for the attenuation compensation in OWM. To overcome the limitations of OWM, two-way wave equations are employed in reverse-time migration (RTM) (McMechan, 1983; and Whitmore, 1983). RTM is capable of imaging interfaces with steep dips and turning waves. Fletcher et al. (2012) separately applied phase and amplitude filters post modeling to the source and receiver wavefields in the frequency domain in RTM. Based on the dispersion relationship in a linear viscoacoustic medium (Kjartansson, 1979), Zhang et al. (2010) derived a pseudo-differential equation to model wave propagation and applied it with regularization for backward propagation in RTM. Suh et al. (2012) extended this method to compensate for the viscoacoustic attenuation effects in anisotropic media.

Given a Q model, a viscoelastic mechanical model consisting of standard linear solids (SLSs) provides a powerful tool to model real earth materials (Robertsson et al., 1994). One SLS consists of a spring in parallel with a spring and a dashpot in series. It can approximate a constant Q within a defined frequency band. A series of SLSs connected in parallel can yield a quite general mechanical viscoelasticity (Day and Minster, 1984). In an SLS the stress-strain relationship is expressed as a causal time convolution of a stress relaxation function with the strain rate. Given a Q model, a viscoelastic mechanical model consisting of standard linear solids (SLSs) provides a powerful tool to model real earth materials (Robertsson et al., 1994). One SLS consists of a spring in parallel with a spring and a dashpot in series. It can approximate a constant Q within a defined frequency band. A series of SLSs connected in parallel can yield a quite general mechanical viscoelasticity (Day and Minster, 1984). In an SLS the stress-strain relationship is expressed as a causal time convolution of a stress relaxation function with the strain rate. Liu et al. (1976; Day and Minster, 1984; Carcione, 1993; Robertsson et al., 1994). This time dependence of the relaxation mechanism is governed by the stress and strain relaxation times. They describe the physical dissipation mechanism that the real earth materials have on wave propagation. Finite-difference wavefield extrapolations implemented on staggered grids have shown that the SLSs can simulate wave propagation well (Robertsson et al., 1994; Larsen et al., 1998). Based on a single SLS, Bai et al. (2012 and 2013) derived a viscoacoustic wave equation for forward modeling and its adjoint, and solved them by finite-different methods on centered grids to estimate velocity and Q models in viscoacoustic waveform inversion. In the equations memory variables simulate viscosity.

We attempted to use the forward viscoacoustic wave equation derived by Bai et al. (2012) and its backward wave equation for viscoacoustic RTM. However, the backward propagation of the receiver wavefield is unstable because we do not have the recorded data for the memory variable. Instead, we now derive a new viscoacoustic wave equation without any memory variable. The equation includes a pseudo-differential operator in the time-space domain for modeling the viscosity. Although the approach is similar to the one proposed by Zhang et al. (2010), the pseudo-differential operator is straightforward to implement. Following the discussion of Suh et al. (2012), we implement the lossy acoustic wave equations for both forward and backward wave propagation to compensate for the attenuation effects in RTM. A numerical test indicates that the attenuation effects can be properly compensated.
We also provide a real data example to show the compensation for the attenuation effects caused by low-Q anomalies in complex geological structures.

**Theory**

In a viscoacoustic model consisting of one SLS, a viscoacoustic wave equation can be used to simulate the attenuation effects on seismic waves during their propagation (Bai et al., 2012).

\[
\frac{1}{v^2} \frac{\partial^2 P}{\partial t^2} = (1 + \tau) \nabla \cdot \nabla P - r + f \tag{1}
\]

with

\[
r = \frac{\tau}{\tau_s} \left[ e^{-\tau_s H(t)} \right] \ast [\nabla \cdot \nabla P],
\]

where \( P = P(x,t,x_s) \) is the wavefield at time \( t \) and at a position \( x \) for a source located at \( x_s \), \( v = v(x) \) is velocity, \( f \) is the source, \( H(t) \) is the Heaviside function, and \( r \) is a memory variable. The memory variable is a causal time convolution and describes the dissipation mechanism. Its kernel is of exponential character. Since it decays, energy is dissipated. The stress relaxation time \( \tau_s \) and the strain relaxation time \( \tau_r \) can be calculated from \( Q \) over a frequency band (Carcione, 2001).

\[
\tau_s = \frac{\sqrt{Q^2 + 1} - 1}{\omega Q} \tag{3}
\]

and

\[
\tau_r = \frac{\sqrt{Q^2 + 1} + 1}{\omega Q}, \tag{4}
\]

where \( \omega \) is the angular frequency. In equation 2 \( \tau \) is a unitless variable defined as \( \tau = \tau_r / \tau_s \). Since \( Q \) is rarely less than 20 for any real materials (Robertsson et al., 1994), it is assumed that \( \tau << 1 \). Blanch et al. (1995) indicated that \( \tau \) is slightly too small due to the approximation.

Without the source term the equations 1 and 2 lead to the following dispersion relationship

\[
\frac{\partial^2 P}{\partial t^2} = \frac{k}{v} \left[ \frac{1}{1 + \omega^2 \tau_r^2} \right] \nabla \cdot \nabla P \tag{5}
\]

where \( v_0 \) is a constant velocity for the Fourier analysis. This relationship is consistent with the one described by Carcione et al. (1988) for an array of SLSs. The dispersion relationship can be re-written as

\[
\frac{\omega^2}{v} = k \left[ \frac{1 + \omega \tau_r \tau_s}{1 + \omega^2 \tau_r^2} + i \frac{\omega \tau_s}{1 + \omega^2 \tau_r^2} \right]. \tag{6}
\]

It is useful to get the following approximation from equations 3 and 4

\[
1 + \omega^2 \tau_r^2 \approx 1. \tag{7}
\]

With the assumption \( \tau << 1 \), we can easily prove that

\[
\frac{\omega \tau_r}{v^2} \approx \frac{\tau}{2}. \tag{8}
\]

Equations 5, 7 and 8 give us the following dispersion relationship

\[
\frac{\omega^2}{v} = k \left( 1 + i \frac{\tau}{2} \right). \tag{9}
\]

As a result, equation 9 leads to the relationship between \( |k| \) and \( \omega \)

\[
|k| = \frac{\omega}{v} \left( 1 - i \frac{\tau}{4} \right). \tag{10}
\]

After some algebra, we eventually have the relationship

\[
\omega^2 - i \omega \frac{\tau_r}{v} |k| = v^2 k^2 = 0. \tag{11}
\]

In the \( t-x \) domain this dispersion relationship is written as

\[
\frac{\partial^2}{\partial t^2} + \frac{\tau_r}{v} \sqrt{-\Delta} \frac{\partial}{\partial t} - v^2 k^2 P = 0, \tag{12}
\]

where the second term is a pseudo-differential operator and \( \Delta \) is the Laplacian operator. There is no memory variable in the equation. The second term is a lossy term and the third term is a dispersive term. Following the discussion of Suh et al. (2012), we have the following lossy acoustic wave equation if we care only for the attenuation.

\[
\frac{\partial^2}{\partial t^2} + \frac{\tau_r}{v} \sqrt{-\Delta} \frac{\partial}{\partial t} - v^2 k^2 P = 0, \tag{13}
\]

where “+” sign is used for the forward propagation and “-” sign for the backward propagation. Wavefield extrapolation is performed for a 2D 2800x2760 m model with a constant velocity (= 1500m/s) and a constant \( Q (= 20) \). A Ricker wavelet with the peak frequency of 18 Hz is used. Snapshots are obtained from the acoustic wave equation (Figure 1(a)), the viscoacoustic wave equation 1 (Figure 1(b)) and the lossy acoustic wave equation 13 (Figure 1(c)). Figure 1(d) indicates that the viscoacoustic equation 1 produces a phase shift plus amplitude attenuation relative to the acoustic wave equation, while the lossy equation 13 does not have the phase shift.

In viscoacoustic RTM we use the cross-correlation imaging condition. A high-cut wavenumber filter is applied to the pseudo-differential operator to stabilize the backward propagation of the recorded wavefield data.
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Figure 1: Snapshots obtained from (a) the acoustic wave equation, (b) the viscoacoustic wave equation 1, and (c) the lossy acoustic wave equation 13. (d) Traces extracted from panel (a) (blue), panel (b) (red) and panel (c) (black) at x = 1400 m. Note that the panel (c) preserves phase.

Example

First a 2D viscoacoustic model is used to test this method. The velocity and Q models are displayed in Figure 2(a) and 2(b), respectively. In the models both a low velocity anomaly and a low Q anomaly are present in the trapezoidal-shaped area. The low-value Q means strong attenuation in the trapezoid. Viscoacoustic synthetic shot gathers are generated using a Ricker wavelet with a peak frequency of 18 Hz, with a total of 151 shots. The shot interval is 100 m. Each shot has 661 receivers. The receiver interval is 15 m. The image is shown in Figure 2(c) when the synthetic data is migrated with acoustic RTM. As expected, the effects of the low Q anomaly are seen in a dimming of amplitudes from the reflectors beneath the anomaly. The attenuation effects are compensated (Figure 2(d)) when the data is migrated with viscoacoustic RTM.

Next we show a real data example. A wiband workflow is applied to generate a broadband seismic data (Zhou et al., 2012). The broadband data is migrated with frequencies up to 60 Hz to generate high-resolution images for comparison. From a viscoacoustic waveform inversion a Q model is obtained from an initial Q model in which the Q value is 5000. The high-value Q means no attenuation at the beginning. Figure 3 shows the inverted Q model overlaying the migrated image without any Q compensation. The low Q anomalies cause low amplitudes along reflectors beneath them. In Figure 4 we compare the migrations from acoustic RTM and viscoacoustic RTM in the area of interest shown in Figure 3. Figure 5 indicates that the viscoacoustic RTM enhances the high-wavenumber components.

Conclusions

Based on the theory of one SLS, we derive the viscoacoustic wave equation, which includes a pseudo-differential operator for the viscosity. The viscoacoustic wave equation has no memory variable. We implement the lossy acoustic wave equations for the forward and backward wave propagation to correct the attenuation effects in RTM. A numerical test demonstrates that the attenuation effects in RTM can be properly compensated. A real data example shows that the attenuation compensation enhances the high-wavenumber components in complex geological structures.

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Figure 2: A synthetic example. (a) Velocity model. Velocity unit is m/s. (b) Q model. Images obtained from (c) acoustic RTM and (d) viscoacoustic RTM.

Figure 3: Q model overlays the image from acoustic RTM. The color bar indicates the range of low Q anomalies.

Figure 4: Images obtained from (a) acoustic RTM and (b) viscoacoustic RTM.

Figure 5: Amplitude spectra obtained from the box shown in Figure 4(a). Blue curve is obtained from acoustic RTM and red curve from viscoacoustic RTM. The wavenumber is $k_z$. 
EDITED REFERENCES

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