Practical Strategies for Waveform Inversion
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Introduction
The term “waveform inversion” refers to a collection of techniques that use the information from the times and waveform shapes of seismic data to derive high fidelity velocity models for seismic imaging. Waveform inversion was first introduced by Lailly, Tarantola, and Mora (Lailly, 1983; Tarantola, 1984; Mora, 1988). Since these pioneering efforts many researchers have attempted to use various strategies and computational schemes to make waveform inversion implemented either in the time or the frequency domain a processing tool for real data sets.

The attractiveness of waveform inversion lies mainly in its lack of approximations, at least in a formal theoretical sense, in contrast to other traditional velocity determination techniques such as semblance or tomography. However, a whole raft of approximations must be made to make the technique viable with today’s computing technology and restrictions of seismic data acquisition. Some of these approximations are rather severe, such as restriction to acoustic waveform inversion while others are made simply to speed up the process. These are collectively referred to as “waveform inversion strategies”, which turn the whole process into a very manpower intensive art form.

This paper discusses these various strategies and their influences on the velocity models that are obtained from waveform inversion. One cannot exhaustively test all the choices of strategies, and for that reason the paper focuses on the choices that in our experience create the most difficulties. These approaches will be illustrated on data from offshore Brazil.

Fundamental Choice of strategies
Waveform inversion techniques seek to minimize the misfit between the seismic data $d_0$ and the data $d$ modeled through some scheme, usually a finite difference acoustic algorithm. The pros and cons of using time or frequency domain techniques and various details of implementations have been discussed for example in Yingst et al (2011).

Currently most practitioners limit the waveform inversion to the acoustic approximation. This is a pragmatic choice, dictated by the available computing power currently available. It is fair to say that no one has fully assessed the effect of this assumption on the derived velocity models. However as is well known, large AVO effects exists in shallow sections caused in large part by contrasts in shear velocities. Also some data sets have prominent converted waves, which must be accounted for separately. This must be kept in mind because as has been seen in several examples a high frequency component is imprinted in the derived velocity models, which is usually attributed to unaccounted for elastic variations. This can be a severe problem when inverting land data.

A more severe restriction is the assumption of constant density. Again this is usually a pragmatic restriction, usually related to the fact that wave equation solvers are designed for seismic imaging, where a constant density assumption is benign. Not so in waveform inversion however. In tertiary basins across the world the contrast at the water bottom is mostly caused by the large variation between water and sediment densities. The compressional velocity contrast at the water bottom is usually negligible in these geological areas. This causes a problem, since the modelled data will be missing one of the most prominent reflection events, namely the water bottom. Various strategies are employed to circumvent this problem such as muting the water bottom and starting the inversion “slightly below” the water bottom. Although the incorporation of a variable density is not a fundamental issue, it would involve estimating another parameter in the model, or simply fixing the density as a known function of velocity. None of these techniques are completely satisfactory and the pragmatic approach as been the muting approach discussed above.

Finally absorption/attenuation effects are also usually neglected, or fixed using some pre-determined $Q$ parameter. Whereas the latter option is usually preferable, in either case the changing of the bandwidth with depth causes serious misties between the real data and the modelled data. This is usually handled interactively by changing the wavelet with depth for the modelled data.

Synthetics versus real data
Waveform inversion on real data derives most of its information from “diving waves”, and typically less from...
reflection data. However, many papers perform various tests and design strategies using synthetic data sets. For synthetic data sets, waveform inversion is able to derive a

**Figure 1** (a) Synthetic velocity model. (b) Model inverted from diving waves. (c) Model from reflections.

**Figure 2** Ray paths from single shot in the initial velocity model. Maximum offset is 10000 m and diving energy shuts off at a depth of ~5000 m.
reliable velocity model using only reflected data as is shown in Figure 1. The exquisite control that synthetic afford in terms of being able to match the “data” to the model is the fundamental reason for that result. Not so for real data sets, where the variability of wavelets, and of model versus reality frustrates efforts to derive velocity models by inverting reflection data.

In contrast, waveform inversion on real data is usually accomplished by using diving waves that sample the velocity field very well because of their long travel paths in the sediments. But reliance on diving waves means that waveform inversion is restricted to data sets where a strong enough gradient exist to produce turning waves or refracted events, long offsets, and therefore also to shallow updates. Figure 2 shows a model based on a real data set, where the turning waves shut off at ~5000 m due to restricted maximum offsets (10000 m), and shows that updated velocity is reliable up to that depth. We strongly recommend that models such as that in the figure be generated routinely before embarking on a waveform inversion project. Another related issue is that the input data should not mute direct arrivals, which are the first events that are removed in conventional processing.

Cycle skipping, bandwidth and source delay

These effects are closely related in time domain waveform inversion. Frequency domain inversion facilitates strategies that help resolve these issues (Yingst, 2011). Time domain implementations of waveform inversion use frequency bands, usually reasonably narrow. Because of the highly non linear process, and the complexity of the misfit functions, a multi-scale strategy [Bunks et al., 1995] is usually implemented, starting with the lowest frequencies available in the seismic data and increasing the bandwidth. Most seismic data sets lack very low frequencies, and usually begin around 5 Hz. Whereas inversions can be done with that data, it is preferable to have frequencies as low as 2Hz.

Since the inversion is trying to match modelled wave shapes to real data, the wavelet that one assumes for modelling is critical. The larger the bandwidth the more complex the wavelet! Usual consideration forces one to match wavelets to at least half a cycle, which becomes harder with higher frequencies. A tell-tale sign of wavelet mismatch is the appearance of strong banding in the computed gradient, and the lack on convergence of the algorithm.

A closely related but separate effect is the source time delay. Again wavelets used for the modelled data must match the real wavelet “locally”, i.e. where the inversion is performed. Time shifts between these two wavelets greater than half a cycle will cause a strong banding effect in the gradient and a lack of convergence. Note that this source time delay will change with depth, a therefore

a natural strategy which will be discussed shortly is to “layer strip”, so that the source time delay can be optimized in shorter time windows in addition to carefully managing the offset range.

Layer stripping, masking and offset weighting

A commonly used technique for waveform inversion is layer stripping: the velocity is only inverted in a determined layer, flat or conforming to the geology using interpreted horizons. A closely related strategy is to simply mask entire regions that need not be updated such as the water column, or the reflection at the water bottom. The layer stripping approach speeds up the entire process by reducing the computational burden within each iteration. It also affords an easier “local” determination of the processing parameters. The layer can be chosen to optimize where the diving energy is dominant within the data set. The layer stripping approach also allows processors to determine the source delay time only within a small time window. A common issue with the layer stripping approach is the necessity to taper the edges of the strip, to avoid numerical instabilities and artefacts. Unfortunately, the taper lengths and smoothness are not easily determined by theory and must be determined experimentally.

Another strategy commonly used is offset weighting which lets the user focus the inversion of the parts of the data that are most affected by the velocity speeding up convergence. The type of offset weighting is left to experimentation. Figure 3 shows the difference in derived gradients using different offset weights. Experimentation and practice are required to choose best offset weights. Offset weighting is also used to separate reflected versus refracted energy. Weighting can also be applied in other domains such as reflection angle domain. Which weighting is best is very much data dependent.
Brazil example

An example from offshore Brazil is shown. This is a 2D Brazil Span line acquired in 2008 in the area of Santos Basin near Lula Oilfield. The selected 2D line was acquired with offsets up to 10,200m. No pre-processing has been applied to the input data used for waveform inversion. The geometry of the line assumes 2D with no cable feathering and with preservation of source-receiver offset.

The source signature was derived from the data. No debubble or zero phase filters were applied to the wavelet nor to the input data. Far offset weighting was used for the waveform inversion. Low frequencies below 10hz were used in the inversion.

An initial inversion shows an improvement in the top of salt image as shown in figure 4 (Kirchhoff migrated stack using an initial sediment model from production on the left. On the right is the migrated stack using a model after waveform inversion). Figure 5 shows a migrated stack volume on the left using a velocity model obtained from waveform inversion. On the right is the velocity obtained from waveform inversion. The red box shows the zoomed area shown in figure 4. The turquoise box shows the zoomed area shown in figure 6. Figure 6 shows a zoomed area from each migrated stack, before inversion on the left and after waveform inversion on the right. The difference in the stacks is hard to see but the difference between starting model and resultant model show subtle differences in the velocity after waveform inversion. We believe this to be due to geology. The left side of figure 7 shows the zoomed area of the migrated stack after inversion with a velocity difference overlay. The right shows gathers migrated with the updated model obtained from waveform inversion.

Conclusions

This paper presents a different aspect of waveform inversion, namely the sensitivities of the resulting velocity models on the various strategies utilized during an inversion project. The paper focuses only on the most important issues in the authors’ experience. The practice of waveform inversion depends crucially on mastering these strategies.

References


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**Figure 4** Migrated stacks before (left) and after (right) waveform inversion.

**Figure 5** Migrated stack and velocity model after waveform inversion.
Figure 6 Migrated stacks before (left) and after (right) waveform inversion (zoomed area).

Figure 7 Left: Migrated stack after waveform inversion with velocity difference overlay (zoomed area).
Right: Migrated gathers after waveform inversion (zoomed area).